Pensieve header: Verifying the w-Alexander link relations produced by wAlexanderLinkRelations.m using pA.

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\2010-08"];
<< wAlexanderLinkRelations.m
<< pA.m
```

Some further pA values from WSofMVA.nb:

```
pA[P[i_, j_]] := AHD[
   t[i] == t[j], {i}, W[j],
   W[j] - W[i]
   ];
pA[SVXp[i_, j_, k_, l_]] := AHD[
   (t[i] == t[k]) (t[j] == t[l]), {i, j}, W[k, l],
   -W[i, k] + t[j] W[i, k] + W[i, l] - t[i] W[i, l] - W[k, l] + t[i] W[k, l]
   ];
pA[B[i_, j_]] := AHD[
   (t[i] == t[j]), {i}, W[j], W[i] - t[i] W[i] - W[j] + t[i] W[j]
   ];
```

```
wJD[wjd_wJD] := wjd;
wJD[s_Plus] := wJD /@ s;
wJD[c_ ? NumberQ * expr_] := c wJD[expr];
wJD[p_Times] := Join @@ (wJD @@ (wJD /@ p));
wJD[(el_) ^ (k_)] := wJD @@ Table[el, {k}];
pW[n_, wjd_wJD] := Module[
    {multiperms, i, j, alldirs, pdirs, ks, p, ti, tj},
    multiperms = Table[
      Permutations[Range[Count[wjd, i, Infinity]]],
      {i, n}
     ];
    If[MemberQ[multiperms, {{}}], 0,
     alldirs = Flatten[Outer[pdirs, Sequence @@ multiperms, 1]];
     Expand[
      Sum[
         ks = Table[0, {n}];
         Last[pA[Join[
            (CircuitDiagram @@ wjd) /. {
              x[i_] :> (
                ti = 10 i + p[[i, ++ks[[i]]]];
                B[ti - 1, ti]
               ),
              a[i_, j_] :> (
                ti = 10 i + p[[i, ++ks[[i]]]];
                tj = 10 j + p[[j, ++ks[[j]]]];
                SVXp[ti - 1, tj - 1, ti, tj]
               )
             },
            CircuitDiagram @@ Table[P[10 i + ks[[i]], 10 i], {i, 2, n}]
           ]]],
         {p, alldirs}
        ] /. {W[10] → 1, W[_] → 0} /. t[i_] :> x[i / 10] + 1
      ]
     ]
   ];
pW[n_, expr_] := Expand[expr /. wjd_wJD :> pW[n, wjd]]

{s0, s1} = {1, 1};
rrels = wJD /@ RRels[2, 1];
Table[pW[2, rrels[[k]]], {k, Length[rrels]}]

{0}

{s0, s1} = {-1, -1};
rrels = wJD /@ RRels[2, 1];
Table[pW[2, rrels[[k]]], {k, Length[rrels]}]

{0}

{s0, s1} = {1, 1};
rrels = wJD /@ RRels[2, 2];
Table[pW[2, rrels[[k]]], {k, Length[rrels]}]

{0, 4 x[1] x[2], 0, 0, 0, 0, 0}
```

```
{s0, s1} = {-1, -1};
rrels = wJD /@ RRels[2, 2];
Table[pW[2, rrels[[k]]], {k, Length[rrels]}]
```

$\{0, 0, 0, 0, 0, 0, 0\}$

```
{s0, s1} = {1, -1};
rrels = wJD /@ RRels[2, 2];
Table[pW[2, rrels[[k]]], {k, Length[rrels]}]
```

$\{0, 0, 0, -2 x[1] x[2], 0, 0, 0, 0\}$

```
{s0, s1} = {-1, 1};
rrels = wJD /@ RRels[2, 2];
Table[pW[2, rrels[[k]]], {k, Length[rrels]}]
```

$\{0, 0, 0, -2 x[1] x[2], 0, 0, 0, 0\}$

```
{s0, s1} = {1, 1};
rrels = wJD /@ RRels[2, 3];
Table[pW[2, rrels[[k]]], {k, Length[rrels]}]
```

$\{0, 0, 0, 12 x[1]^2 x[2], 12 x[1]^2 x[2], 0, 24 x[1] x[2]^2, 0, 8 x[1]^2 x[2],$
$0, 0, -6 x[1] x[2]^2, 0, 0, 0, 0, 0, 0, 0, 0, 0, -6 x[1] x[2]^2, 0, 0, 0, 0\}$

```
{s0, s1} = {-1, -1};
rrels = wJD /@ RRels[2, 3];
Table[pW[2, rrels[[k]]], {k, Length[rrels]}]
```

$\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}$

```
{s0, s1} = {1, -1};
rrels = wJD /@ RRels[2, 3];
Table[pW[2, rrels[[k]]], {k, Length[rrels]}]
```

$\{0, -6 x[1]^2 x[2], 0, 0, -6 x[1]^2 x[2], 0, 0, 0, 0, 0, -6 x[1]^2 x[2], -12 x[1]^2 x[2], 0,$
$-6 x[1] x[2]^2, 0, 0, -12 x[1] x[2]^2, 0, 0, 0, -4 x[1]^2 x[2], 0, 0, 0, 0, -6 x[1] x[2]^2, 0, 0, 0, 0\}$

```
{s0, s1} = {-1, 1};
rrels = wJD /@ RRels[2, 3];
Table[pW[2, rrels[[k]]], {k, Length[rrels]}]
```

$\{0, -6 x[1]^2 x[2], 0, 0, -6 x[1]^2 x[2], 0, 0, 0, 0, 0, -6 x[1]^2 x[2], -12 x[1]^2 x[2], 0,$
$-6 x[1] x[2]^2, 0, 0, -12 x[1] x[2]^2, 0, 0, 0, -4 x[1]^2 x[2], 0, 0, 0, 0, -6 x[1] x[2]^2, 0, 0, 0, 0\}$

```
{s0, s1} = {-1, -1};
rrels = wJD /@ RRels[3, 2];
Table[pW[3, rrels[[k]]], {k, Length[rrels]}]
```

$\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}$

```
{s0, s1} = {1, 1};
rrels = wJD /@ RRels[3, 2];
Table[pW[3, rrels[[k]]], {k, Length[rrels]}]
```

$\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}$

```
{s0, s1} = {-1, -1};
rrels = wJD /@ RRels[3, 3];
Table[pW[3, rrels[[k]]], {k, Length[rrels]}]
```

{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}

```
{s0, s1} = {1, 1};
rrels = wJD /@ RRels[3, 3];
Table[pW[3, rrels[[k]]], {k, Length[rrels]}]
```

$\{0, 0, 0, 0, 0, 0, 8\,x[1]^2\,x[2], 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,$
 $8\,x[1]\,x[2]\,x[3], 0, 0, 0, 8\,x[1]^2\,x[3], 0, 0, 16\,x[1]\,x[2]\,x[3], 0, 0, 0, 0, 0, 0, 0, 0,$
 $0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -6\,x[1]\,x[2]^2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,$
 $0, 0, 0, 0, 0, 0, 0, 0, 0, 8\,x[1]\,x[2]\,x[3], 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,$
 $0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 8\,x[1]\,x[2]\,x[3], 0, 0, 0, 0, 0,$
 $0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -6\,x[1]\,x[3]^2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,$
 $0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}$

```
{s0, s1} = {-1, -1};
pWRank[m_] := Module[
   {mons, mat, im},
   mons = Table[x[1]^i x[2]^(m - i), {i, 0, m}];
   mat = DeleteCases[
       Sort[
        pW[2, #] & /@ (wJD /@ Gens[2, m])
        ],
        0
      ] /. Thread[mons → IdentityMatrix[m + 1]];
   {
    im = DeleteCases[RowReduce[mat].mons, 0],
    Length[im]
   }
  ]

Table[
 Print[m → (r = pWRank[m])]; r,
 {m, 4}
 ]
```

$1 \to \{\{x[1]\}, 1\}$

$2 \to \{\{x[1]\,x[2], x[1]^2\}, 2\}$

$3 \to \{\{x[1]\,x[2]^2, x[1]^2\,x[2], x[1]^3\}, 3\}$

$4 \to \{\{x[1]\,x[2]^3, x[1]^2\,x[2]^2, x[1]^3\,x[2], x[1]^4\}, 4\}$

```
$Aborted

Print[# → pW[2, wJD[#]]] & /@ Gens[2, 1]; RRels[2, 1]
```

a[1, 1] → 0

a[1, 2] → -x[1]

a[2, 1] → 0

a[2, 2] → 0

x[1] → 0

x[2] → 0

{x[2]}

```
Print[# → pW[2, wJD[#]]] & /@ Gens[2, 2]; RRels[2, 2]
```

$a[1, 1]^2 \to 0$

$a[1, 1] a[1, 2] \to -3 x[1]^2$

$a[1, 2]^2 \to -4 x[1]^2$

a[1, 1] a[2, 1] → 0

a[1, 2] a[2, 1] → -2 x[1] x[2]

$a[2, 1]^2 \to 0$

a[1, 1] a[2, 2] → 0

a[1, 2] a[2, 2] → -3 x[1] x[2]

a[2, 1] a[2, 2] → 0

$a[2, 2]^2 \to 0$

a[1, 1] x[1] → 0

$a[1, 2] x[1] \to -2 x[1]^2$

a[2, 1] x[1] → 0

a[2, 2] x[1] → 0

$x[1]^2 \to 0$

a[1, 1] x[2] → 0

a[1, 2] x[2] → -2 x[1] x[2]

a[2, 1] x[2] → 0

a[2, 2] x[2] → 0

x[1] x[2] → 0

$x[2]^2 \to 0$

$\{a[2, 1] x[1], a[2, 2] x[1], a[1, 1] x[2], a[2, 1] x[2], a[2, 2] x[2], x[1] x[2], x[2]^2\}$

```
Print[# → pW[2, wJD[#]]] & /@ Gens[2, 3]; RRels[2, 3]
```

$a[1, 1]^3 \to 0$

$a[1, 1]^2 a[1, 2] \to -40 x[1]^3$

$a[1, 1] a[1, 2]^2 \to -24 x[1]^3$

$a[1, 2]^3 \rightarrow -36\, x[1]^3$

$a[1, 1]^2\, a[2, 1] \rightarrow 0$

$a[1, 1]\, a[1, 2]\, a[2, 1] \rightarrow -16\, x[1]^2\, x[2]$

$a[1, 2]^2\, a[2, 1] \rightarrow -18\, x[1]^2\, x[2]$

$a[1, 1]\, a[2, 1]^2 \rightarrow 0$

$a[1, 2]\, a[2, 1]^2 \rightarrow -12\, x[1]\, x[2]^2$

$a[2, 1]^3 \rightarrow 0$

$a[1, 1]^2\, a[2, 2] \rightarrow 0$

$a[1, 1]\, a[1, 2]\, a[2, 2] \rightarrow -9\, x[1]^2\, x[2]$

$a[1, 2]^2\, a[2, 2] \rightarrow -32\, x[1]^2\, x[2]$

$a[1, 1]\, a[2, 1]\, a[2, 2] \rightarrow 0$

$a[1, 2]\, a[2, 1]\, a[2, 2] \rightarrow -12\, x[1]\, x[2]^2$

$a[2, 1]^2\, a[2, 2] \rightarrow 0$

$a[1, 1]\, a[2, 2]^2 \rightarrow 0$

$a[1, 2]\, a[2, 2]^2 \rightarrow -40\, x[1]\, x[2]^2$

$a[2, 1]\, a[2, 2]^2 \rightarrow 0$

$a[2, 2]^3 \rightarrow 0$

$a[1, 1]^2\, x[1] \rightarrow 0$

$a[1, 1]\, a[1, 2]\, x[1] \rightarrow -12\, x[1]^3$

$a[1, 2]^2\, x[1] \rightarrow -12\, x[1]^3$

$a[1, 1]\, a[2, 1]\, x[1] \rightarrow 0$

$a[1, 2]\, a[2, 1]\, x[1] \rightarrow -6\, x[1]^2\, x[2]$

$a[2, 1]^2\, x[1] \rightarrow 0$

$a[1, 1]\, a[2, 2]\, x[1] \rightarrow 0$

$a[1, 2]\, a[2, 2]\, x[1] \rightarrow -6\, x[1]^2\, x[2]$

$a[2, 1]\, a[2, 2]\, x[1] \rightarrow 0$

$a[2, 2]^2\, x[1] \rightarrow 0$

$a[1, 1]\, x[1]^2 \rightarrow 0$

$a[1, 2]\, x[1]^2 \rightarrow -6\, x[1]^3$

$a[2, 1]\, x[1]^2 \rightarrow 0$

$a[2, 2]\, x[1]^2 \rightarrow 0$

$x[1]^3 \rightarrow 0$

$a[1, 1]^2\, x[2] \rightarrow 0$

$a[1, 1]\, a[1, 2]\, x[2] \rightarrow -6\, x[1]^2\, x[2]$

$a[1, 2]^2\, x[2] \rightarrow -12\, x[1]^2\, x[2]$

a[1, 1] a[2, 1] x[2] → 0

a[1, 2] a[2, 1] x[2] → -6 x[1] x[2]^2

a[2, 1]^2 x[2] → 0

a[1, 1] a[2, 2] x[2] → 0

a[1, 2] a[2, 2] x[2] → -12 x[1] x[2]^2

a[2, 1] a[2, 2] x[2] → 0

a[2, 2]^2 x[2] → 0

a[1, 1] x[1] x[2] → 0

a[1, 2] x[1] x[2] → -4 x[1]^2 x[2]

a[2, 1] x[1] x[2] → 0

a[2, 2] x[1] x[2] → 0

x[1]^2 x[2] → 0

a[1, 1] x[2]^2 → 0

a[1, 2] x[2]^2 → -6 x[1] x[2]^2

a[2, 1] x[2]^2 → 0

a[2, 2] x[2]^2 → 0

x[1] x[2]^2 → 0

x[2]^3 → 0

$\Big\{$a[1, 1] a[2, 1] x[1], a[1, 2] a[2, 1] x[1] - a[1, 1] a[1, 2] x[2], a[2, 1]^2 x[1],

a[1, 1] a[2, 2] x[1], a[1, 2] a[2, 2] x[1] $- \dfrac{1}{2}$ a[1, 2]^2 x[2], a[2, 1] a[2, 2] x[1],

a[2, 2]^2 x[1], a[2, 1] x[1]^2, a[2, 2] x[1]^2, a[1, 1]^2 x[2], a[1, 1] a[2, 1] x[2], a[2, 1]^2 x[2],

a[1, 1] a[2, 2] x[2], a[2, 1] a[2, 2] x[2], a[2, 2]^2 x[2], a[1, 1] x[1] x[2], a[2, 1] x[1] x[2],

a[2, 2] x[1] x[2], x[1]^2 x[2], a[1, 1] x[2]^2, a[2, 1] x[2]^2, a[2, 2] x[2]^2, x[1] x[2]^2, x[2]^3$\Big\}$

Print[# → pW[2, wJD[#]]] & /@ Gens[2, 4]; RRels[2, 4]

a[1, 1]^4 → 0

a[1, 1]^3 a[1, 2] → -1260 x[1]^4

a[1, 1]^2 a[1, 2]^2 → -480 x[1]^4

a[1, 1] a[1, 2]^3 → -360 x[1]^4

a[1, 2]^4 → -576 x[1]^4

a[1, 1]^3 a[2, 1] → 0

a[1, 1]^2 a[1, 2] a[2, 1] → -360 x[1]^3 x[2]

a[1, 1] a[1, 2]^2 a[2, 1] → -240 x[1]^3 x[2]

a[1, 2]^3 a[2, 1] → -288 x[1]^3 x[2]

a[1, 1]^2 a[2, 1]^2 → 0

$a[1, 1] \, a[1, 2] \, a[2, 1]^2 \to -180 \, x[1]^2 \, x[2]^2$

$a[1, 2]^2 \, a[2, 1]^2 \to -192 \, x[1]^2 \, x[2]^2$

$a[1, 1] \, a[2, 1]^3 \to 0$

$a[1, 2] \, a[2, 1]^3 \to -144 \, x[1] \, x[2]^3$

$a[2, 1]^4 \to 0$

$a[1, 1]^3 \, a[2, 2] \to 0$

$a[1, 1]^2 \, a[1, 2] \, a[2, 2] \to -120 \, x[1]^3 \, x[2]$

$a[1, 1] \, a[1, 2]^2 \, a[2, 2] \to -192 \, x[1]^3 \, x[2]$

$a[1, 2]^3 \, a[2, 2] \to -540 \, x[1]^3 \, x[2]$

$a[1, 1]^2 \, a[2, 1] \, a[2, 2] \to 0$

$a[1, 1] \, a[1, 2] \, a[2, 1] \, a[2, 2] \to -96 \, x[1]^2 \, x[2]^2$

$a[1, 2]^2 \, a[2, 1] \, a[2, 2] \to -240 \, x[1]^2 \, x[2]^2$

$a[1, 1] \, a[2, 1]^2 \, a[2, 2] \to 0$

$a[1, 2] \, a[2, 1]^2 \, a[2, 2] \to -120 \, x[1] \, x[2]^3$

$a[2, 1]^3 \, a[2, 2] \to 0$

$a[1, 1]^2 \, a[2, 2]^2 \to 0$

$a[1, 1] \, a[1, 2] \, a[2, 2]^2 \to -120 \, x[1]^2 \, x[2]^2$

$a[1, 2]^2 \, a[2, 2]^2 \to -720 \, x[1]^2 \, x[2]^2$

$a[1, 1] \, a[2, 1] \, a[2, 2]^2 \to 0$

$a[1, 2] \, a[2, 1] \, a[2, 2]^2 \to -240 \, x[1] \, x[2]^3$

$a[2, 1]^2 \, a[2, 2]^2 \to 0$

$a[1, 1] \, a[2, 2]^3 \to 0$

$a[1, 2] \, a[2, 2]^3 \to -1260 \, x[1] \, x[2]^3$

$a[2, 1] \, a[2, 2]^3 \to 0$

$a[2, 2]^4 \to 0$

$a[1, 1]^3 \, x[1] \to 0$

$a[1, 1]^2 \, a[1, 2] \, x[1] \to -240 \, x[1]^4$

$a[1, 1] \, a[1, 2]^2 \, x[1] \to -120 \, x[1]^4$

$a[1, 2]^3 \, x[1] \to -144 \, x[1]^4$

$a[1, 1]^2 \, a[2, 1] \, x[1] \to 0$

$a[1, 1] \, a[1, 2] \, a[2, 1] \, x[1] \to -80 \, x[1]^3 \, x[2]$

$a[1, 2]^2 \, a[2, 1] \, x[1] \to -72 \, x[1]^3 \, x[2]$

$a[1, 1] \, a[2, 1]^2 \, x[1] \to 0$

$a[1, 2] \, a[2, 1]^2 \, x[1] \to -48 \, x[1]^2 \, x[2]^2$

$a[2, 1]^3 \, x[1] \to 0$

$a[1, 1]^2 a[2, 2] x[1] \rightarrow 0$

$a[1, 1] a[1, 2] a[2, 2] x[1] \rightarrow -36 x[1]^3 x[2]$

$a[1, 2]^2 a[2, 2] x[1] \rightarrow -96 x[1]^3 x[2]$

$a[1, 1] a[2, 1] a[2, 2] x[1] \rightarrow 0$

$a[1, 2] a[2, 1] a[2, 2] x[1] \rightarrow -36 x[1]^2 x[2]^2$

$a[2, 1]^2 a[2, 2] x[1] \rightarrow 0$

$a[1, 1] a[2, 2]^2 x[1] \rightarrow 0$

$a[1, 2] a[2, 2]^2 x[1] \rightarrow -80 x[1]^2 x[2]^2$

$a[2, 1] a[2, 2]^2 x[1] \rightarrow 0$

$a[2, 2]^3 x[1] \rightarrow 0$

$a[1, 1]^2 x[1]^2 \rightarrow 0$

$a[1, 1] a[1, 2] x[1]^2 \rightarrow -60 x[1]^4$

$a[1, 2]^2 x[1]^2 \rightarrow -48 x[1]^4$

$a[1, 1] a[2, 1] x[1]^2 \rightarrow 0$

$a[1, 2] a[2, 1] x[1]^2 \rightarrow -24 x[1]^3 x[2]$

$a[2, 1]^2 x[1]^2 \rightarrow 0$

$a[1, 1] a[2, 2] x[1]^2 \rightarrow 0$

$a[1, 2] a[2, 2] x[1]^2 \rightarrow -18 x[1]^3 x[2]$

$a[2, 1] a[2, 2] x[1]^2 \rightarrow 0$

$a[2, 2]^2 x[1]^2 \rightarrow 0$

$a[1, 1] x[1]^3 \rightarrow 0$

$a[1, 2] x[1]^3 \rightarrow -24 x[1]^4$

$a[2, 1] x[1]^3 \rightarrow 0$

$a[2, 2] x[1]^3 \rightarrow 0$

$x[1]^4 \rightarrow 0$

$a[1, 1]^3 x[2] \rightarrow 0$

$a[1, 1]^2 a[1, 2] x[2] \rightarrow -80 x[1]^3 x[2]$

$a[1, 1] a[1, 2]^2 x[2] \rightarrow -72 x[1]^3 x[2]$

$a[1, 2]^3 x[2] \rightarrow -144 x[1]^3 x[2]$

$a[1, 1]^2 a[2, 1] x[2] \rightarrow 0$

$a[1, 1] a[1, 2] a[2, 1] x[2] \rightarrow -48 x[1]^2 x[2]^2$

$a[1, 2]^2 a[2, 1] x[2] \rightarrow -72 x[1]^2 x[2]^2$

$a[1, 1] a[2, 1]^2 x[2] \rightarrow 0$

$a[1, 2] a[2, 1]^2 x[2] \rightarrow -48 x[1] x[2]^3$

$a[2, 1]^3 x[2] \rightarrow 0$

$a[1, 1]^2 a[2, 2] x[2] \rightarrow 0$

$a[1, 1] a[1, 2] a[2, 2] x[2] \rightarrow -36 x[1]^2 x[2]^2$

$a[1, 2]^2 a[2, 2] x[2] \rightarrow -160 x[1]^2 x[2]^2$

$a[1, 1] a[2, 1] a[2, 2] x[2] \rightarrow 0$

$a[1, 2] a[2, 1] a[2, 2] x[2] \rightarrow -60 x[1] x[2]^3$

$a[2, 1]^2 a[2, 2] x[2] \rightarrow 0$

$a[1, 1] a[2, 2]^2 x[2] \rightarrow 0$

$a[1, 2] a[2, 2]^2 x[2] \rightarrow -240 x[1] x[2]^3$

$a[2, 1] a[2, 2]^2 x[2] \rightarrow 0$

$a[2, 2]^3 x[2] \rightarrow 0$

$a[1, 1]^2 x[1] x[2] \rightarrow 0$

$a[1, 1] a[1, 2] x[1] x[2] \rightarrow -24 x[1]^3 x[2]$

$a[1, 2]^2 x[1] x[2] \rightarrow -36 x[1]^3 x[2]$

$a[1, 1] a[2, 1] x[1] x[2] \rightarrow 0$

$a[1, 2] a[2, 1] x[1] x[2] \rightarrow -18 x[1]^2 x[2]^2$

$a[2, 1]^2 x[1] x[2] \rightarrow 0$

$a[1, 1] a[2, 2] x[1] x[2] \rightarrow 0$

$a[1, 2] a[2, 2] x[1] x[2] \rightarrow -24 x[1]^2 x[2]^2$

$a[2, 1] a[2, 2] x[1] x[2] \rightarrow 0$

$a[2, 2]^2 x[1] x[2] \rightarrow 0$

$a[1, 1] x[1]^2 x[2] \rightarrow 0$

$a[1, 2] x[1]^2 x[2] \rightarrow -12 x[1]^3 x[2]$

$a[2, 1] x[1]^2 x[2] \rightarrow 0$

$a[2, 2] x[1]^2 x[2] \rightarrow 0$

$x[1]^3 x[2] \rightarrow 0$

$a[1, 1]^2 x[2]^2 \rightarrow 0$

$a[1, 1] a[1, 2] x[2]^2 \rightarrow -18 x[1]^2 x[2]^2$

$a[1, 2]^2 x[2]^2 \rightarrow -48 x[1]^2 x[2]^2$

$a[1, 1] a[2, 1] x[2]^2 \rightarrow 0$

$a[1, 2] a[2, 1] x[2]^2 \rightarrow -24 x[1] x[2]^3$

$a[2, 1]^2 x[2]^2 \rightarrow 0$

$a[1, 1] a[2, 2] x[2]^2 \rightarrow 0$

$a[1, 2] a[2, 2] x[2]^2 \rightarrow -60 x[1] x[2]^3$

$a[2, 1] a[2, 2] x[2]^2 \rightarrow 0$

$a[2, 2]^2 x[2]^2 \rightarrow 0$

$a[1, 1] \, x[1] \, x[2]^2 \to 0$

$a[1, 2] \, x[1] \, x[2]^2 \to -12 \, x[1]^2 \, x[2]^2$

$a[2, 1] \, x[1] \, x[2]^2 \to 0$

$a[2, 2] \, x[1] \, x[2]^2 \to 0$

$x[1]^2 \, x[2]^2 \to 0$

$a[1, 1] \, x[2]^3 \to 0$

$a[1, 2] \, x[2]^3 \to -24 \, x[1] \, x[2]^3$

$a[2, 1] \, x[2]^3 \to 0$

$a[2, 2] \, x[2]^3 \to 0$

$x[1] \, x[2]^3 \to 0$

$x[2]^4 \to 0$

$$\left\{ a[1, 1]^2 \, a[2, 1] \, x[1], \; a[1, 1] \, a[1, 2] \, a[2, 1] \, x[1] - a[1, 1]^2 \, a[1, 2] \, x[2], \right.$$

$$a[1, 2]^2 \, a[2, 1] \, x[1] - a[1, 1] \, a[1, 2]^2 \, x[2], \; a[1, 1] \, a[2, 1]^2 \, x[1],$$

$$a[1, 2] \, a[2, 1]^2 \, x[1] - a[1, 1] \, a[1, 2] \, a[2, 1] \, x[2], \; a[2, 1]^3 \, x[1],$$

$$a[1, 1]^2 \, a[2, 2] \, x[1], \; a[1, 1] \, a[1, 2] \, a[2, 2] \, x[1] - \frac{1}{2} a[1, 1] \, a[1, 2]^2 \, x[2],$$

$$a[1, 2]^2 \, a[2, 2] \, x[1] - \frac{2}{3} a[1, 2]^3 \, x[2], \; a[1, 1] \, a[2, 1] \, a[2, 2] \, x[1],$$

$$a[1, 2] \, a[2, 1] \, a[2, 2] \, x[1] - a[1, 1] \, a[1, 2] \, a[2, 2] \, x[2], \; a[2, 1]^2 \, a[2, 2] \, x[1],$$

$$a[1, 1] \, a[2, 2]^2 \, x[1], \; a[1, 2] \, a[2, 2]^2 \, x[1] - \frac{1}{2} a[1, 2]^2 \, a[2, 2] \, x[2], \; a[2, 1] \, a[2, 2]^2 \, x[1],$$

$$a[2, 2]^3 \, x[1], \; a[1, 1] \, a[2, 1] \, x[1]^2, \; a[1, 2] \, a[2, 1] \, x[1]^2 - a[1, 1] \, a[1, 2] \, x[1] \, x[2],$$

$$a[2, 1]^2 \, x[1]^2, \; a[1, 1] \, a[2, 2] \, x[1]^2, \; a[1, 2] \, a[2, 2] \, x[1]^2 - \frac{1}{2} a[1, 2]^2 \, x[1] \, x[2],$$

$$a[2, 1] \, a[2, 2] \, x[1]^2, \; a[2, 2]^2 \, x[1]^2, \; a[2, 1] \, x[1]^3, \; a[2, 2] \, x[1]^3, \; a[1, 1]^3 \, x[2],$$

$$a[1, 1]^2 \, a[2, 1] \, x[2], \; a[1, 2]^2 \, a[2, 1] \, x[2] - 2 \, a[1, 1] \, a[1, 2] \, a[2, 2] \, x[2], \; a[1, 1] \, a[2, 1]^2 \, x[2],$$

$$a[2, 1]^3 \, x[2], \; a[1, 1]^2 \, a[2, 2] \, x[2], \; a[1, 1] \, a[2, 1] \, a[2, 2] \, x[2], \; a[2, 1]^2 \, a[2, 2] \, x[2],$$

$$a[1, 1] \, a[2, 2]^2 \, x[2], \; a[2, 1] \, a[2, 2]^2 \, x[2], \; a[2, 2]^3 \, x[2], \; a[1, 1]^2 \, x[1] \, x[2],$$

$$a[1, 1] \, a[2, 1] \, x[1] \, x[2], \; a[1, 2] \, a[2, 1] \, x[1] \, x[2] - a[1, 1] \, a[1, 2] \, x[2]^2, \; a[2, 1]^2 \, x[1] \, x[2],$$

$$a[1, 1] \, a[2, 2] \, x[1] \, x[2], \; a[1, 2] \, a[2, 2] \, x[1] \, x[2] - \frac{1}{2} a[1, 2]^2 \, x[2]^2, \; a[2, 1] \, a[2, 2] \, x[1] \, x[2],$$

$$a[2, 2]^2 \, x[1] \, x[2], \; a[1, 1] \, x[1]^2 \, x[2], \; a[2, 1] \, x[1]^2 \, x[2], \; a[2, 2] \, x[1]^2 \, x[2],$$

$$x[1]^3 \, x[2], \; a[1, 1]^2 \, x[2]^2, \; a[1, 1] \, a[2, 1] \, x[2]^2, \; a[2, 1]^2 \, x[2]^2, \; a[1, 1] \, a[2, 2] \, x[2]^2,$$

$$a[2, 1] \, a[2, 2] \, x[2]^2, \; a[2, 2]^2 \, x[2]^2, \; a[1, 1] \, x[1] \, x[2]^2, \; a[2, 1] \, x[1] \, x[2]^2,$$

$$\left. a[2, 2] \, x[1] \, x[2]^2, \; x[1]^2 \, x[2]^2, \; a[1, 1] \, x[2]^3, \; a[2, 1] \, x[2]^3, \; a[2, 2] \, x[2]^3, \; x[1] \, x[2]^3, \; x[2]^4 \right\}$$